International Journal of Mathematics Research.
ISSN 0976-5840 Volume 10, Number 1 (2018), pp. 7-20

http://www.irphouse.com

Some New Results on Super Stolarsky-3 Mean Labeling of Graphs

S.S. Sandhya
Department of Mathematics, SreeAyyappa College for Women, Chunkankadai– 629003, Tamilnadu, India,

E. Ebin Raja Merly
Department of Mathematics, Nesamony Memorial Christian College, Marthandam – 629165, Tamilnadu, India,

S.Kavitha
Department of Mathematics, Holy Cross College, Nagercoil- 629 004 , Tamilnadu, India,

Abstract

Here we discuss some new results on Super Stolarsky-3 Mean Labeling of graphs .In this paper, we prove that Cycle, Flag graph, Dumbbell graph and Kayak paddle graphs are Super Stolarsky-3 mean labeling of graphs.
Key words*:* Super Stolarsky-3 Mean labeling, Flag graph, Dumbbell graph and Kayak paddle graph.

1. INTRODUCTION

The graph considered here will be simple, finite and undirected graph G= (V, E) with p vertices and q edges without loops or parallel edges. For all detailed survey of graph labeling, we refer to J.A.Gallian [1]. For all other terminology and notations we follow Harary [2].

The following definitions are necessary for our present investigation.

Definition 1.1: Let G = (V,E) be a graph with p vertices and q edges.
Let f: V (G)→{1,2,.., p+q} be an injective function. For a vertex labeling f, the induced edge labeling f* (e=uv) is defined by

f* (e) == $\left\lceil \sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}} \right\rceil$ (or) $\left\lfloor \sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}} \right\rfloor$. Then f is called a Super Stolarsky-3 Mean labeling if f(V(G))∪{f(e) / e ϵ E(G)} = {1, 2, …, p + q}.
A graph which admits Super Stolarsky-3 Mean labeling is called Super Stolarsky-3 Mean graphs.

Definition 1.2: A closed path is called a cycle. A cycle on n vertices is denoted by C_n.

Definition 1.3: The Flag graph Fl_n is obtained by joining one vertex of Cn to an extra vertex is called the root.

Definition 1.4: The Dumbbell graphs D(n,m) is obtained by joining two disjoint cycles C_n *and* C_m with an edge..

Definition 1.5: Kayak Paddle KP (n,m,t) is the graph obtained by joining C_n and C_m by a path of length t.

2. MAIN RESULTS

Theorem 2.1: Any Cycle is Super Stolarsky-3 Mean graph.

Proof: Here we consider two cases.

Case (i) n is odd

Let C_n be the cycle of length n and $u_1, u_2, \dots, u_n$ be the vertices and u_1u_2, u_iu_{i+2}, i=2,4,6,….,n-3, $u_{n-1}u_n$, u_iu_{i+2}, i=1,3,5,….,n-2 be the edges of C_n.

Define a function **f** : V(C_n)→{ 1,2,…., p+q} by

$$f(u_1) = 1.$$

$$f(u_i) = 2i, \quad i=2,4,6,\dots,n-1.$$

$f(u_3) = f(u_1) + 4.$

$f(u_i) = f(u_{i-2}) + 4$, i=5,7,9,….,n-2.

$f(u_n) = f(u_{n-1}) + 2.$

Then the edge labels are distinct.

Case (ii) n is even

Let C_n be the cycle of length n and $u_1, u_2, \dots, u_n$ be the vertices and u_1u_2, u_iu_{i+2}, i=2,4,6,….,n-2, $u_{n-1}u_n$, $u_{n-3}u_{n-1}$, u_iu_{i+1}, i=1,3,5,….,n-5 be the edges of C_n.

Define a function **f** : $V(C_n) \rightarrow \{1,2,\dots, p+q\}$ by

$f(u_1) = 1.$

$f(u_i) = 2i$, i=2,4,6,….,n.

$f(u_3) = f(u_1) + 4.$

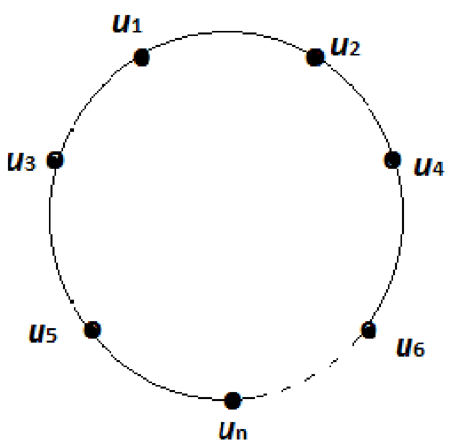
$f(u_i) = f(u_{i-2}) + 4$, i=5,7,9,….,n-1.

In this case also we get the edge labels are distinct.

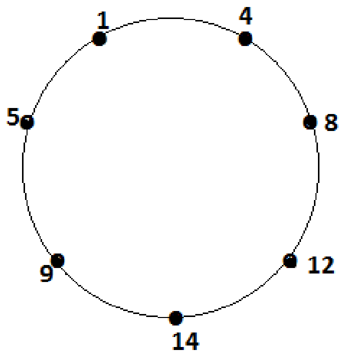
Hence Cycle C_n is a Super Stolarsky-3 Mean graph.

Example 2.2:

The Super Stolarsky-3 Mean labeling of C_n is given below.



The following figure shows Super Stolarsky-3 Mean labeling of C_7 and C_8.



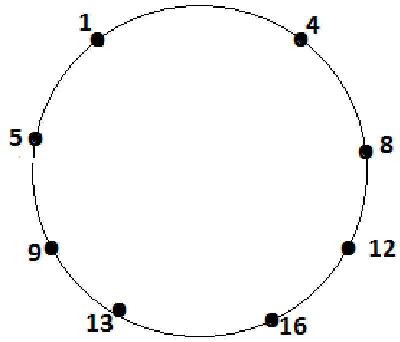


Figure: 1

Theorem 2.3: The Flag graph Fl_n is Super Stolarsky-3 Mean graph if n ≥ 3.

Proof: Let Fl_n be a Flag graphs.

Here we consider two cases.

Case (i) n is odd

Let $u_0, u_1, u_2, \dots, u_n$ be the vertices and u_1u_2, u_iu_{i+2}, i=2,4,6,….,n-3, $u_{n-1}u_n$, u_iu_{i+2}, i=1,3,5,….,n-2 , u_nu_0 be the edges of Fl_n.

Define a function **f** : V(Fl_n)→{ 1,2,…., p+q} by

$$f(u_1) = 1.$$

$$f(u_i) = 2i, \quad i=2,4,6,\dots, n-1.$$

$$f(u_3) = f(u_1) + 4.$$

$$f(u_i) = f(u_{i-2}) + 4, \quad i=5,7,9,\dots,n-2.$$

$$f(u_n) = f(u_{n-1}) + 2.$$

$$f(u_0) = f(u_n) + 2.$$

Then the edge labels are distinct.

Case (ii) n is even

Let $u_0, u_1, u_2, \dots, u_n$ be the vertices and u_1u_2, u_iu_{i+2}, i=2,4,6,….,n-2, $u_{n-1}u_n$, u_iu_{i+2}, i=1,3,5,….,n-3 , u_nu_0 be the edges of Fl_n.

Define a function **f** : V(Fl_n)→{ 1,2,...., p+q} by

$f(u_1) = 1.$

$f(u_i) = 2i, \quad i=2,4,6,....,n.$

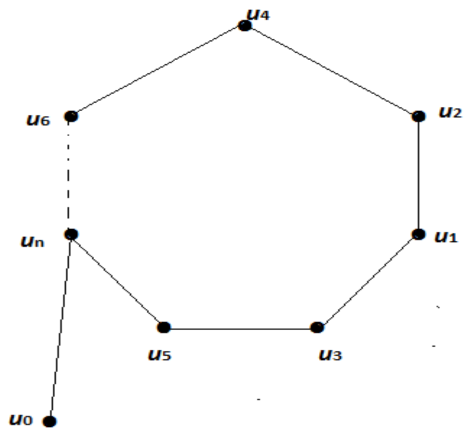
$f(u_3) = f(u_1) + 4.$

$f(u_i) = f(u_{i-2}) + 4, \; i=5,7,9,....,n-1.$

$f(u_0) = f(u_n) + 2.$

From Case (i) and case (ii), we conclude that Flag graph Fl_n is Super Stolarsky-3 Mean graph.

Example 2.4: Super Stolarsky-3 Mean Labeling of Flag graph Fl_n is given below.



Super Stolarsky-3 Mean Labeling of Flag graph Fl_7 and Fl_6 is given below.

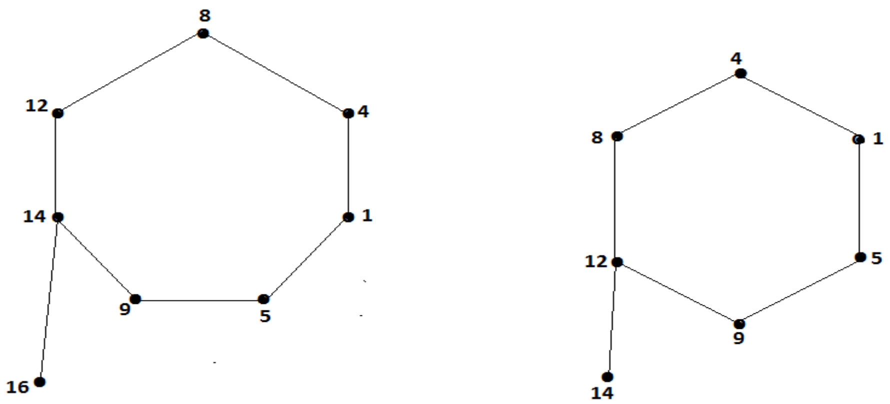


Figure: 2

Theorem 2.5: The Dumbbell graph D (n,m) is Super Stolarsky-3 Mean graph if n, m$\geq$ 3.

Proof: Let D (n,m) be a Dumbbell graph. Consider the following cases.

Case (i) n is even and m is even

Let $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_m$ be the vertices and u_1u_2, u_iu_{i+2}, i=2,4,6,….,n-2, $u_{n-1}u_n$, u_iu_{i+2}, i=1,3,5,….,n-3 , u_nv_1 , v_1v_2, v_iv_{i+2}, i=2,4,6,….,n-2, v_iv_{i+2}, i =1,3,5,….,n-3, $v_{n-1}v_n$ be the edges of D(n,m).

Define a function **f** : V(D(n,m)) → { 1,2,…., p+q} by

$f(u_1)$ =1.

$f(u_i)$ =2i, i=2,4,6,….,n.

$f(u_3) = f(u_1)$ +4.

$f(u_i) = f(u_{i-2}) + 4$, i=5,7,9,….,n-1.

$f(v_1) = f(u_n)$ +2.

$f(v_2)$= $f(v_1)$ +4.

$f(v_3)$= $f(v_1)$ +3.

$f(v_i) = f(v_{i-2}) + 4$, i=4,5,6,7,….,m-1.

$f(v_m)$= $f(v_{m-1})$ +4.

Then the edge labels are distinct.

Case (ii) n is odd and m is odd

Let $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_m$ be the vertices and u_1u_2, u_iu_{i+2}, i=2,4,6,….,n-3, $u_{n-1}u_n$, u_iu_{i+2}, i=1,3,5,….,n-2 , u_nv_1 , v_1v_2, v_iv_{i+2}, i=2,4,6,….,n-3, v_iv_{i+2}, i =1,3,5,….,n-2, $v_{n-1}v_n$ be the edges of D(n,m).

Define a function **f** : V(D(n,m)) → { 1,2,…., p+q} by

$f(u_1)$ =1.

$f(u_i)$ =2i, i=2,4,6,….,n-1.

$f(u_3) = f(u_1)$ +4.

$f(u_i) = f(u_{i-2}) + 4$, i=3,5,7,9,….,n-2.

f(u_n) = f(u_{n-1}) +2.

f(v_1) = f(u_n) +2.

f(v_2)= f(v_1) +4.

f(v_3)= f(v_1) +3.

f(v_i) = f(v_{i-2}) + 4, i=4,5,6,7,….,m, i≠m-1

f(v_{m-1})= f(v_{m-3}) +5.

Then the edge labels are distinct.

Case (iii) n is even and m is odd

Let $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_m$ be the vertices and u_1u_2, u_iu_{i+2}, i=2,4,6….,n-2, $u_{n-1}u_n$, u_iu_{i+2}, i=1,3,5,….,n-3 , u_nv_1 , v_1v_2, v_iv_{i+2}, i=2,4,6,….,n-2, v_iv_{i+2}, i =1,3,5,….,n-3, $v_{n-1}v_n$ be the edges of D(n,m).

Define a function **f** : V(D(n,m)) → { 1,2,…., p+q} by

f(u_1) =1.

f(u_i) =2i, i=2,4,6,….,n.

f(u_3) = f(u_1) +4.

f(u_i) = f(u_{i-2}) + 4, i=5,7,9,….,n-1.

f(v_1) = f(u_n) +2.

f(v_2)= f(v_1) +4.

f(v_3)= f(v_1) +3.

f(v_i) = f(v_{i-2}) + 4, i=4,5,6,7,….,m-2, i≠ $m-1$.

f(v_{m-1})= f(v_{m-3}) +5.

Then the edge labels are distinct.

Case (iv) n is odd and m is even

Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_m$ be the vertices and $u_1 u_2$, $u_i u_{i+2}$, i=2,4,6,….,n-3, $u_{n-1} u_n$, $u_i u_{i+2}$, i=1,3,5,….,n-2 , $u_n v_1$, $v_1 v_2$, $v_i v_{i+2}$, i=2,4,6,….,n-3, $v_i v_{i+2}$, i =1,3,5,….,n-2, $v_{n-1} v_n$ be the edges of D(n,m).

Define a function **f** : V(D(n,m)) → { 1,2,…., p+q} by

f(u_1) =1.

f(u_i) =2i, i=2,4,6,….,n-1.

f(u_3) = f(u_1) +4.

f(u_i) = f(u_{i-2}) + 4, i=3,5,7,9,….,n-2.

f(u_n) = f(u_{n-1}) +2.

f(v_1) = f(u_n) +2.

f(v_2)= f(v_1) +4.

f(v_3)= f(v_1) +3.

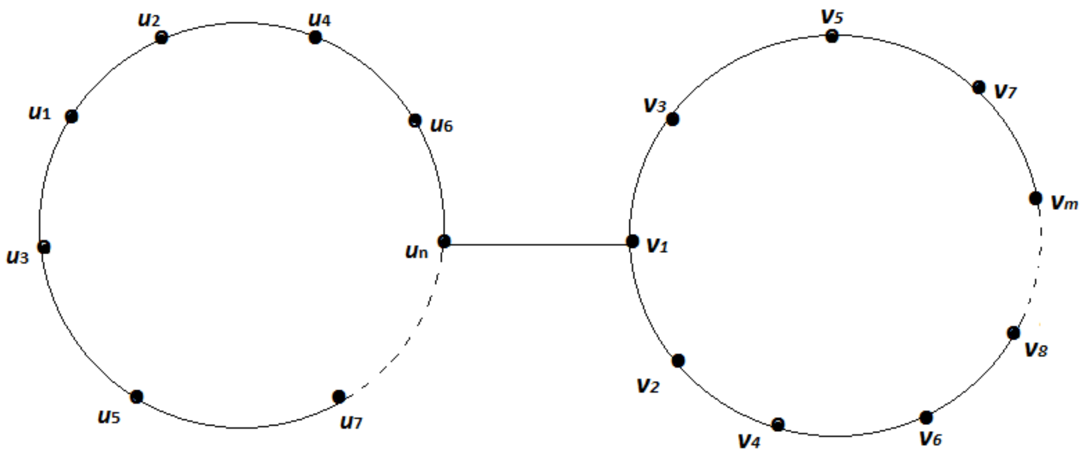
f(v_i) = f(v_{i-2}) + 4, i=4,5,6,7,….,m-1 .

f(v_m)= f(v_{m-1}) +4. f(u_i) =2i, i=2,4,6,….,n-1.

Then the edge labels are distinct.

From case(i),(ii),(iii) and (iv) we conclude that Dumbbell graph D(n,m) is Super Stolarsky-3 Mean graph

Example 2.6: The Stolarsky-3 Mean labeling of D(n,m) is given below.



The following figure shows the Stolarsky-3 Mean labeling of D(8,9) and D(5,6).

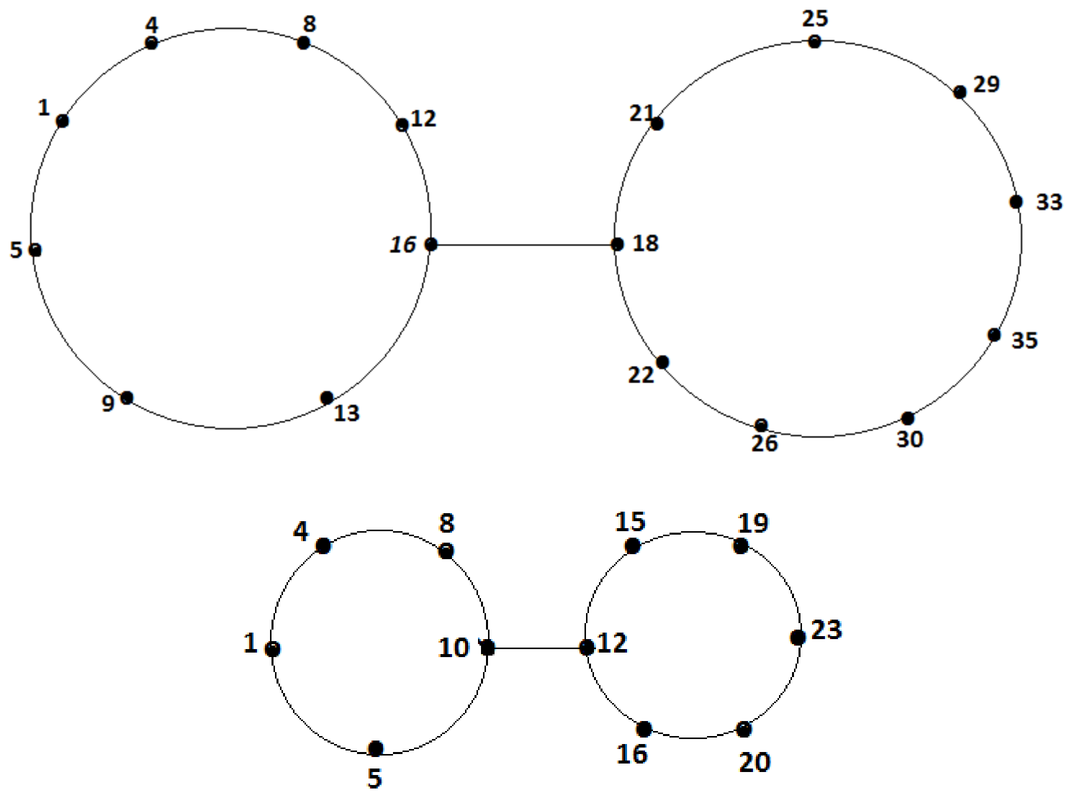


Figure: 3

Theorem 2.7: The Kayak Paddle graph KP (n,m,t) is Super Stolarsky-3 Mean graph.

Proof: Let KP (n,m,t) be Kayak Paddle graph. Consider the following cases.

Case (i) n is even and m is even

Let $u_1, u_2, \dots, u_n$, $v_1, v_2, \dots, v_m$ and $w_1, w_2, \dots, w_t$ be the vertices and u_1u_2, u_iu_{i+2}, i=2,4,6,….,n-2, $u_{n-1}u_n$, u_iu_{i+2}, i=1,3,5,….,n-3, u_nv_1 , v_1v_2, v_iv_{i+2}, i=2,4,6,….,n-2, v_iv_{i+2}, i =1,3,5,….,n-3, $v_{n-1}v_n$, $w_{i-1}w_i$, $1\leq i \leq t$, be the edges of KP(n,m,t).

Define a function **f** : V(KP(n,m,t)) → { 1,2,…., p+q} by

f(u_1) =1.

f(u_i) =2i, i=2,4,6,….,n.

$f(u_3) = f(u_1) + 4$.

$f(u_i) = f(u_{i-2}) + 4$, i=5,7,9,….,n-1.

$f(w_1) = f(u_n)$

$f(w_i) = f(w_{i-1}) + 2$, i=2,3,4,….,t.

$f(v_1) = f(w_t)$.

$f(v_2) = f(v_1) + 4$.

$f(v_3) = f(v_1) + 3$.

$f(v_i) = f(v_{i-2}) + 4$, i=4,5,6,7,….,m-1.

$f(v_m) = f(v_{m-1}) + 4$.

Then the edge labels are distinct.

Case (ii) n is odd and m is odd

Let $u_1, u_2, \ldots, u_n$, $v_1, v_2, \ldots, v_m$ and $w_1, w_2, \ldots, w_t$ be the vertices and u_1u_2, u_iu_{i+2}, i=2,4,6,….,n-3, $u_{n-1}u_n$, u_iu_{i+2}, i=1,3,5,….,n-2, u_nv_1, v_1v_2, v_iv_{i+2}, i=2,4,6,….,n-3, v_iv_{i+2}, i=1,3,5,….,n-2, $v_{n-1}v_n$, $w_{i-1}w_i$, $1 \leq i \leq t$, be the edges of KP(n,m,t).

Define a function **f** : V(KP(n,m,t)) → { 1,2,…., p+q} by

$f(u_1) = 1$.

$f(u_i) = 2i$, i=2,4,6,….,n-1.

$f(u_3) = f(u_1) + 4$.

$f(u_i) = f(u_{i-2}) + 4$, i=3,5,7,9,….,n-2.

$f(u_n) = f(u_{n-1}) + 2$.

$f(w_1) = f(u_n)$

$f(w_i) = f(w_{i-1}) + 2$, i=2,3,4,….,t.

$f(v_1) = f(w_t)$

$f(v_2) = f(v_1) + 4$.

$f(v_3) = f(v_1) + 3.$

$f(v_i) = f(v_{i-2}) + 4$, i=4,5,6,7,9,….,m , i≠m-1

$f(v_{m-1}) = f(v_{m-3}) + 5.$

Then the edge labels are distinct.

Case (iii) n is even and m is odd

Let $u_1, u_2, \dots, u_n$, $v_1, v_2, \dots, v_m$ and $w_1, w_2, \dots, w_t$ be the vertices and u_1u_2, u_iu_{i+2}, i=2,4,6,….,n-2, $u_{n-1}u_n$, u_iu_{i+2}, i=1,3,5,….,n-3 , u_nv_1 , v_1v_2, v_iv_{i+2}, i=2,4,6,….,n-2, v_iv_{i+2}, i=1,3,5,….,n-3, $v_{n-1}v_n$, $w_{i-1}w_i$, $1 \le i \le t$, be the edges of KP(n,m,t).

Define a function **f** : V(KP(n,m,t)) → { 1,2,…., p+q} by

$f(u_1) = 1.$

$f(u_i) = 2i$, i=2,4,6,….,n.

$f(u_3) = f(u_1) + 4.$

$f(u_i) = f(u_{i-2}) + 4$, i=5,7,9,….,n-1.

$f(w_1) = f(u_n)$

$f(w_i) = f(w_{i-1}) + 2$, i=2,3,4,….,t.

$f(v_1) = f(w_t)$

$f(v_2) = f(v_1) + 4.$

$f(v_3) = f(v_1) + 3.$

$f(v_i) = f(v_{i-2}) + 4$, i=4,5,6,7,9,….,m-2, $i \neq m-1$.

$f(v_{m-1}) = f(v_{m-3}) + 5.$

Then the edge labels are distinct.

Case (iv) n is odd and m is even

Let $u_1, u_2, \dots, u_n$, $v_1, v_2, \dots, v_m$ and $w_1, w_2, \dots, w_t$ be the vertices and u_1u_2, u_iu_{i+2}, i=2,4,6,….,n-3, $u_{n-1}u_n$, u_iu_{i+2}, i=1,3,5,….,n-2 , u_nv_1 , v_1v_2, v_iv_{i+2}, i=2,4,6,….,n-3, v_iv_{i+2}, i=1,3,5,….,n-2, $v_{n-1}v_n$, $w_{i-1}w_i$, $1 \leq i \leq t$, be the edges of KP(n,m,t).

Define a function **f** : V(KP(n,m,t)) → { 1,2,…., p+q} by

f(u_1) =1.

f(u_i) =2i, i=2, 4, 6,….,n-1.

f(u_i) = f(u_{i-2}) + 4, i=3, 5,7,9,….,n-2.

f(u_n) = f(u_{n-1}) +2.

f(w_1) = f(u_n)

f(w_i) = f(w_{i-1}) + 2, i=2,3,4,….,t.

f(v_1) = f(w_t)

f(v_2)= f(v_1) +4.

f(v_3)= f(v_1) +3.

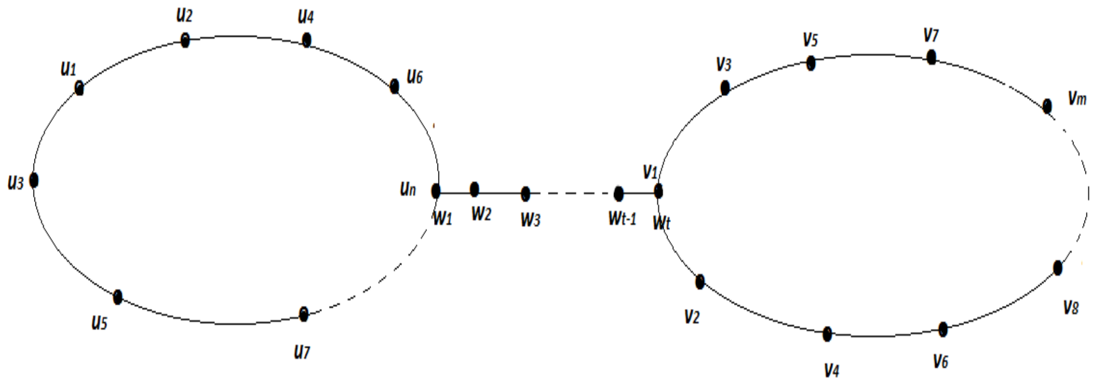
f(v_i) = f(v_{i-2}) + 4, i=4,5,6,7,9,….,m-1 .

f(v_m)= f(v_{m-1}) +4.

Then the edge labels are distinct.

From case(i),(ii), (iii) and (iv), we conclude that Kayak Paddle graph KP(n,m,t) is a Super Stolarsky-3 Mean graph.

Example 2.8: Super Stolarsky-3 Mean labeling of Kayak Paddle graph KP(n,m,t) is shown below.



Super Stolarsky-3 Mean labeling of Kayak Paddle graphs KP(8,9,6) and KP(5,7,5) are shown below.

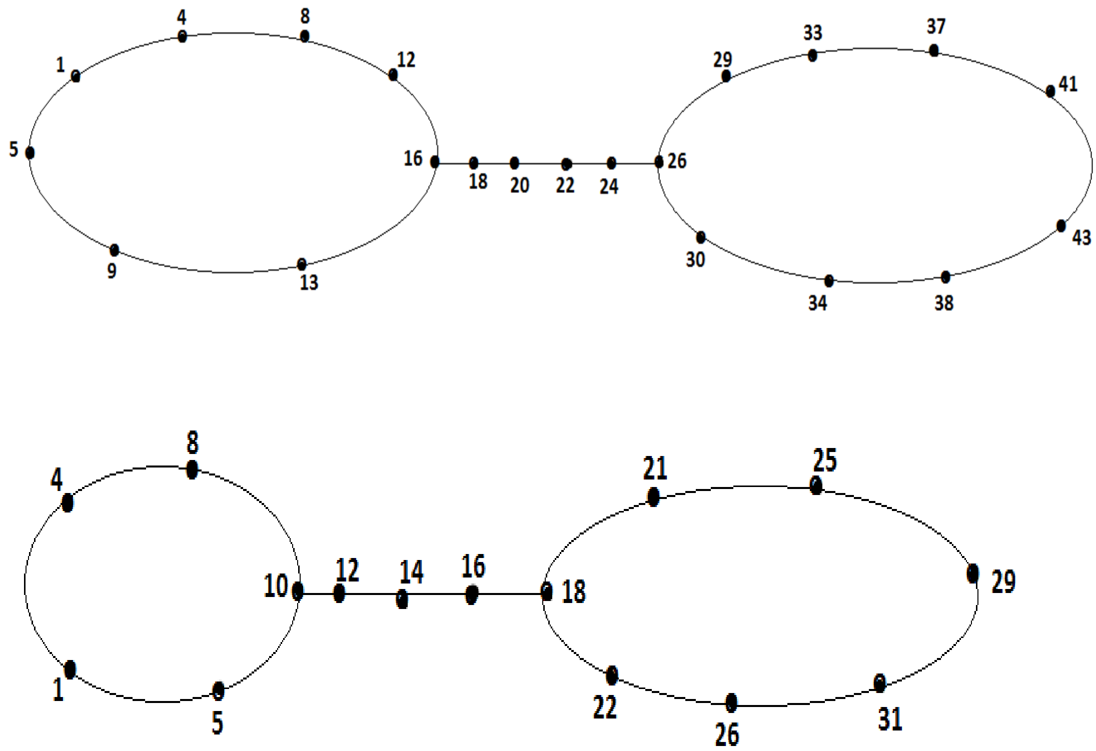


Figure: 4

3. CONCLUSION

In this paper we discussed some new results on Super Stolarsky-3 Mean Labeling of graphs. The authors are of the opinion that the study of Stolarsky-3 Mean labeling behavior of some new graphs using the graph operation shall be quite interesting and also will lead to newer results.

4. ACKNOWLEDGEMENTS

The authors are thank the referees for their valuable comments and suggestions

REFERENCES

[1] J.A. Gallian, "A dynamic survey of graph labeling", The electronic Journal of Combinatories 17(2017),#DS6.

[2] F.Harary, 1988, "Graph Theory" Narosa Puplishing House Reading, New Delhi.

[3] S.Somasundram, R.Ponraj and S.S.Sandhya, " Harmonic Mean Labeling of Graphs" communicated to Journal of Combinatorial Mathematics and combinational computing

[4] V.Hemalatha, V.Mohanaselvi " Super Geometric Mean labeling of some cycle related graphs" -International Journal of Scientific and engineering research, Volume 6, Issue November- 2015 ISSN 2229-5518.

[5] S.S.Sandhya, E. Ebin Raja Merly and S.Kavitha "Stolarsky-3 Mean Labeling of Graphs" Communicated to Journal of discrete Mathematical Sciences and Cryptography.

[6] S.S.Sandhya, E. Ebin Raja Merly and S.Kavitha "Super Stolarsky-3Mean labeling of Some Path Related graphs" Communicated to International Journal of Mathematical combinatorics.